Implementing a Partitioned Algorithm for Fluid-Structure Interaction of Flexible Flapping Wings within *Overture*

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Work Done at

Nanyang Technological University  
Singapore

Sep 21 2010
Outline

- Overture Framework
- Definition of various problems
  - Overlapping grids
  - Fluid dynamics
  - Structural dynamics
  - Rigid body dynamics
- Computational Framework
- Key results and discussions
- Concluding remarks
Overture Framework

Open-Source
Overture Framework
(LLNL Codes)

Overset Moving Grids (ogen)

Deforming Grid 3D + Improvements to Numerical Stability

Parallel Computing on Stationary Grids

A++ Framework Operator Overloading

Interface With PETSc

Overset Flow Solver (OverBlown/cgins)

CSD Solver (cgSD)

FSI
Summary of Work Undertaken using Overture

Rigid and flexible flapping airfoils and wings

Dynamic Stall & Lift Hysteresis
  - Lift Decreases with increase in angle of attack

Vortex dynamics & Thrust Generation
  - How surrounding vorticity field Influences thrust

Passive Flight
  - Accelerating motion of a flapping wing due to Aerodynamic forces

Wing deformation & Fluid Structure Interaction
  - Effect of flexure on Aerodynamic characteristics
Overlapping / Composite / Overset / Chimera Grids

Individual grids can be Generated independently

For 2nd order Scheme: Width of Interpolation = 3 (Quadratic)
Moving Overlapping Grids

Two-Dimensional Moving Grid (Rigid)  Three-Dimensional Deforming Grid
Computational Flow Modeling

(A) Fluid Dynamics – Incompressible Navier Stokes Equations

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\nabla p}{\rho} + \nu \Delta \mathbf{u}
\]

\[\nabla \cdot \mathbf{u} = 0\]

**Pressure Poisson Equation**

\[\Delta p - (\nabla u \cdot \mathbf{u}_x + \nabla v \cdot \mathbf{u}_y + \nabla w \cdot \mathbf{u}_z) = 0\]

- 2\textsuperscript{nd} Order spatial differences
- 2\textsuperscript{nd} Order Crank Nicolson Implicit (For Viscous terms)
- 2\textsuperscript{nd} Order Adams Predictor-Corrector (Explicit)
### Computational Flow Modeling

<table>
<thead>
<tr>
<th>Boundary condition type</th>
<th>Region</th>
<th>Boundary condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall (No slip)</td>
<td>Wing surface</td>
<td>$u = 0$, $\nabla u = 0$, $\frac{\partial p}{\partial n} = n \cdot \left( -\vec{g} - \nu \nabla \times \nabla \times \vec{u} \right)$</td>
</tr>
<tr>
<td>Far field</td>
<td>P-R-R1-P1-P, Q-S-S1-Q1-Q, Q-P-P1-Q1-Q, S-R-R1-S1-S</td>
<td>$n \cdot u = 0$, $\frac{\partial}{\partial n} \left( t_m u \right) = 0$, $\nabla . u = 0$ (Slip wall conditions)</td>
</tr>
<tr>
<td>Inflow</td>
<td>P-Q-S-R-P</td>
<td>$u = u_s$ (velocity specified), $\frac{\partial p}{\partial n} = 0$</td>
</tr>
<tr>
<td>Outflow</td>
<td>P1-Q1-S1-R1-P1</td>
<td>Extrapolate $u$, $\frac{\partial p}{\partial n} = 0$</td>
</tr>
</tbody>
</table>
Computational Modeling

(B) 6 DOF Rigid Body Dynamics

\[ F_A = \text{Net Aerodynamic Force} \]
\[ T = \text{Net Aerodynamic Torque / Moment about Centre of mass (} x_{\text{cms}} \text{)} \]
\[ M = \text{Mass of the body} \]
\[ I = \text{Components of Principal moments of Inertia} \]
\[ \omega = \text{Angular velocity vector} \]
\[ e_i = \text{Principal axes}. \]
\[ \tau = \text{Stress Tensor} \]

\[ F_{A,i} = \int_{d\Omega} p n_i - \tau_{ki} n_k dS \]
\[ M \frac{d^2 x_{\text{cm}}}{dt^2} = F_A \]
\[ I \frac{d\omega}{dt} = T \]
\[ \dot{e}_i = \omega \times e_i \]
\[ I_i \dot{\omega}_i = T \cdot e_i \]
Computational Fluid-Structure Interaction and Coupling Issues

- Rigid body motion of the wing
- Flapping Motion
- Aerodynamic Forces
- Wing deformation due to aerodynamic forces
- Air flowing over the wing
Computational Structural Dynamics Modeling

**Structural Dynamics**

**Kirchhoff’s Plate Equation**

\[
\frac{m_{xz}}{} w_{tt} + D\nabla^4 w = q(x, z, t) \quad D = \frac{Eh^3}{12(1 - \nu^2)}
\]

**Euler-Bernoulli Beam Equation**

\[
m_x \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} = q(x, t)
\]

**Vertical Oscillations**

\[ E : \text{Modulus of Elasticity} \]
\[ h : \text{Plate Thickness} \]
\[ \nu : \text{Poissons Ratio} \]
\[ m_{xz} : \text{Mass per unit area} \]
\[ q : \text{Load Acting} \]
\[ I : \text{Moment of Inertia} \]
Computational Structural Dynamics Modeling

**Structural Dynamics**

\[ w_z = 0 \]

\[ w_{xx} + v w_{zz} = 0 \]
\[ w_{xxx} + (2-v) w_{xzz} = 0 \]

\[ m_{xz} w_{tt} + D \nabla^4 w = q(x, z, t) \]

\[ \nabla^4 w = \sum_{i=1}^{13} A_i w_i = [Q]w, \quad A_i = A, B...M \]

\[ I + \frac{\Delta t^2 D}{m_{xz}} [Q] \]
\[ w^{n+1} = \frac{\Delta t^2 q(x, z, t)^{n+1}}{m_{xz}} + 2w^n - w^{n-1} \]
Discretization of the Euler-Bernoulli Beam Equation:

Airfoil with Flexible Tail

\[ m_x \frac{w_i^{n+1} - 2w_i^n + w_i^{n-1}}{\Delta t^2} + EI \frac{w_{i+2}^{n+1} - 4w_{i+1}^{n+1} + 6w_i^{n+1} - 4w_{i-1}^{n+1} + w_{i-2}^{n+1}}{\Delta x^4} = q(x, t)^{n+1} \]

Solution → \[ [A]y^{n+1} = f(y^n, q^{n+1}) \quad O(\Delta t, \Delta x^2) \]
Fluid – Structure Coupling

Partitioned Approach (*Dirichlet – Neumann Approach*) With Inner Iterations:

\[
\frac{u^p - u^n}{\Delta t} = f(u^{n-1}, u^n, u^p, p^{n-1}, p^n)
\]

\[
\Delta p^p = f(u^p)
\]

\[
[A]u^p = f(u^n, q^{n+1})
\]

Time = $T^n$

Transfer stresses through interpolation

Time = $T^{n+1}$
Load Transfer from Fluid to Solid:

STEP 1

Interpolate Wing Caps on to Wing
Fluid – Structure Coupling

Stress Transfer from Fluid to Solid:

\[ q(x, z, t) = \Pi_u n_u + \Pi_l n_l \]

\[ \Pi = p\delta_{ij} - \tau_{ij} \]
**Fluid – Structure Coupling**

*Comparison of Actual and Interpolated Stress*

**Upper Surface**

- Actual (CFD)
- Interpolated (CSD)

**Lower Surface**

- Actual (CFD)
- Interpolated (CSD)
Fluid – Structure Coupling

Partitioned Approach (Dirichlet – Neumann Approach) With Inner Iterations:

Time = $T^n$

- $\frac{u^p - u^n}{\Delta t} = f(u^{n-1}, u^n, u^p, p^{n-1}, p^n)$
- $\Delta p^p = f(u^p)$
- $[A]w^p = f(w^n, q^{n+1})$

Transfer stresses through interpolation

Local Iterations

- $\frac{u^c - u^n}{\Delta t} = f(u^p, u^n, u^c, p^p, p^n)$
- $\Delta p^c = f(u^c)$
- $[A]w_{k+1}^c = f(w^n, q^c)$

Transfer displacements through interpolation

How to choose $\omega$

$w_{k+1}^c = \omega w_{k+1}^c + (1 - \omega) w_k^c$

$|w_{k+1}^c - w_k^c| < 10^{-9}$

$w^{n+1} = w_{k+1}^c, u^{n+1} = u^c, p^{n+1} = p^c$

Time = $T^{n+1}$
Computational Cases Investigated

Rigid Plunging Wing

Plunging and (active) deforming airfoil

Plunging and (passive) deforming airfoil

Deformation of a beam in a fluid

FSI Coupling Issues

Plunging and passively deforming wing
Motion along Y-axis is given by: 
\[ h = -h_0 \left( 1 - \cos 2\pi f t \right) \quad \text{with} \quad h_0 = 0.175c \]
Reduced frequency, \( k = 0.5, 1.0, 1.82, 2.5, 3.5 \) and 4.0
Reynolds number, \( \text{Re} = 10^4 \)
Rigid Plunging Wing

**Thrust Coefficient**

\[ \bar{C}_T = \frac{2 F_{Ax}}{\rho U_s^2 S C} \]

**Power Input Coefficient**

\[ \bar{C}_{pw} = \frac{2 F_{Ay} \dot{h}}{\rho U_s^3 S C} \]

**Propulsive Efficiency**

\[ \eta = \frac{\bar{C}_T}{\bar{C}_{pw}} \]
Plunging and Deforming (Active) Airfoil

Kinematics:

- **STATIC:** \( y(x, t) = y_0(x) \)
- **RIGID BODY MOTION:** \( y(x, t) = y_0(x) + B(1 - \cos(\omega t)) \)
- **RIGID BODY + DEFORMATION:** \( y(x, t) = y_0(x) + B(1 - \cos(\omega t)) + A(x - x_c)^p \cos(\omega t + Qx + \phi) \)

1. Rigid Plunging: \( B = 0.4, k=2, \text{Re}=10^4, A =0 \) - Tuncer and Kaya (2003)
2. Plunging with Deformation: \( A = 0.3, p = 2, x_c=0, Q=0, \phi=0 \) - Miao and Ho (2006)
Rigid Plunging: \( B = 0.4, k=2, \text{Re}=10^4, A = 0 \) - Tuncer and Kaya (2003)
2. Plunging with Deformation: $A = 0.3$, $p = 2$, $xc=0$, $Q=0$, $\phi=0$, Miao and Ho (2006)
Plunging and Deforming (Active) Airfoil

Miao & Ho (2006)

Present computation

Cp Contours at the mean position
Heathcote et al. (2004)
Tang et al. (2007)

**Rigid** : Length 0.4c, Thickness 0.11c

**Flex** : Length 0.6c, Thickness 0.005c

Re = 9000

\[ E = 2.05 \times 10^{10} \rho U^2 \]

\[ k = f c / U = 1.4 \]

\[ \rho_b = 7.85 \rho \]

Mesh Size : 500 x 11, 200 x 150

---

k = reduced frequency, \( f \) = frequency, \( c \) = beam chord, \( \rho_b \) = beam density, \( \rho \) = Fluid Density, \( U \) = Free-Stream Velocity, \( E \) = Modulus of Elasticity
Effect of Time Step, Relaxation and Damping

Tip Displacement With Net Iteration Index for $\Delta t = 1e-3$

Rigid: Length 0.4c, Thickness 0.11c
Flex: Length 0.6c, Thickness 0.005c

Re $= 9000$
$E = 2.05 \times 10^{10} \rho U^2$
$k = fc/U = 1.4$
$\rho_b = 7.85 \rho$
**Effect of Time Step, Relaxation and Damping**

$\Delta t = 5 \times 10^{-4}$  

$\Delta t = 1 \times 10^{-4}$

Tip Displacement With Net Iteration Index
# Effect of Time Step, Relaxation and Damping

## Table 1. Stability of the fluid-structure interaction problem with a time step of $\Delta t = 10^{-3}$ for various relaxation and numerical damping coefficients

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$\kappa$</th>
<th>0.01</th>
<th>0.015</th>
<th>0.0165</th>
<th>0.0169</th>
<th>0.017</th>
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## Table 2. Stability of the fluid-structure interaction problem with a time step of $\Delta t = 5 \times 10^{-4}$ for various relaxation and numerical damping coefficients

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## Table 3. Stability of the fluid-structure interaction problem with a time step of $\Delta t = 10^{-4}$ for various relaxation and numerical damping coefficients

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Shin et al. (2007)

\[ \frac{b}{c} = 0.003 \]

\[ \frac{\rho_b}{\rho} = 6667 \]

\[ \Gamma = \frac{EI}{\rho_b U^2 bc^2} = 2 \]

\[ \text{Re} = 500 \]

Plate is initially deflected in its first mode for \( \frac{1}{4} \) cycle

\[ b = \text{beam thickness} \]
\[ c = \text{beam chord} \]
\[ \rho_b = \text{beam density} \]
\[ \rho = \text{Fluid Density} \]
\[ U = \text{Free-Stream Velocity} \]

\[ E = \text{Modulus of Elasticity} \]
\[ I = \text{moment of Inertia} \]
Flow Induced Deformation of a Beam

Tip Displacement with Time

Vorticity Contours

Explicit vs Implicit coupling
Heathcote et al. (2008) - Experiments
Aono et al. (2009) - Computations

Re = 10000
E   = 200 Gpa
k   = fc/U = 1.82
ρ_b = 7.85 ρ (Steel Plate)

Typically 20 Inner Iterations (Correction Steps) with a Δt = 2.5x10^{-3} s
Plunging and Deforming Wing

Thrust Coefficient

Experiment
Present Computation

Incompressible NS, nu=1.00e-02
\( t=0.8, d=2.50e-03, \text{ad}T=0.3, \text{ad}z=0.5 \)
A computational framework has been developed to couple fluid dynamics, rigid body dynamics, and structural dynamics

- Developed a partitioned coupling approach for fluid-structure interaction problems
- Importance of relaxation for partitioned coupling approaches
- Coupling an external structural solver with cgins - under progress
Access my Channel @ youtube

http://www.youtube.com/user/2008cfd