

# Chimera Grid Method for Incompressible Flows and its Applications in Actual Problems

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10th Symposium on Overset Composite Grids and  
Solution Technology  
NASA Ames Research Center, CA  
Sept. 20-23, 2010

# Outline

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- I. Introduction
- II. Grid interface algorithms
- III. Incompressible flows
- III. Environmental flows
- IV. Coastal ocean flows
- V. Concluding remarks

## I. Introduction

# Domain decomposition and Chimera grids

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### Advantages

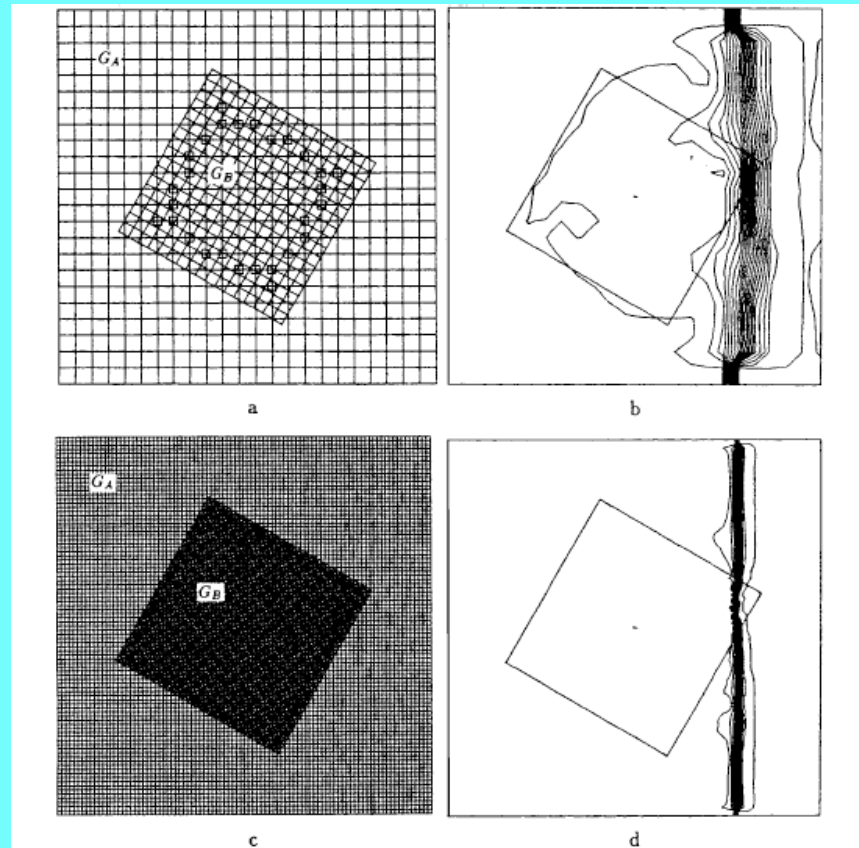
- Overcome difficulty in mesh generation
- Utilize parallel computation approaches
- Attack multi-scale and multi-physics problems

### Grid/Model Interface Treatment --- Crucial Issue

- Conservative algorithms – difficult to realize, unstable, inconsistency, ...
- Non-conservative algorithms – easy to realize, wrong solution, numerical oscillation, ....

## II. Grid Interface Algorithm

# Conservative or non-conservative treatment?



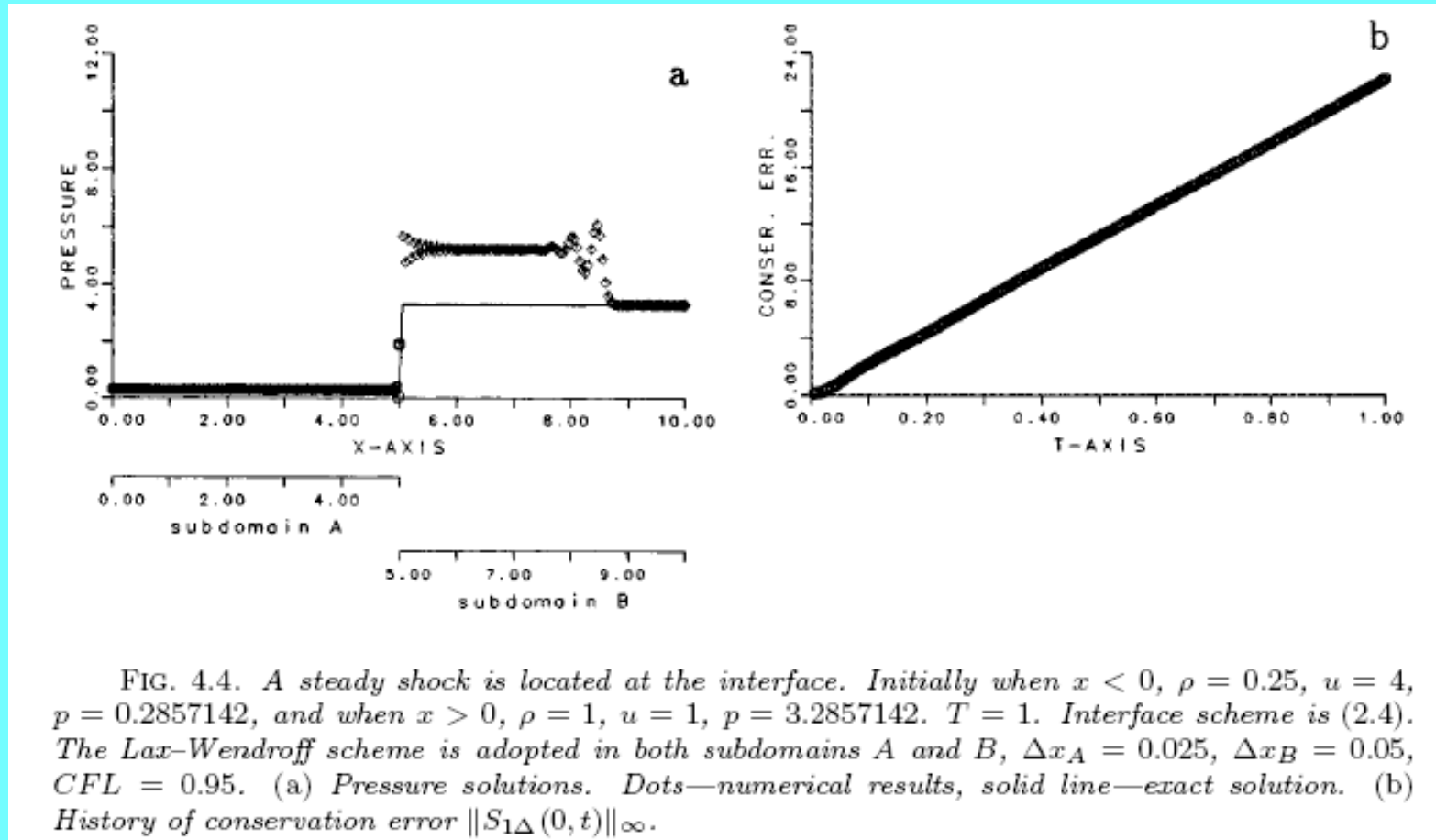
Physical solution obtained with bi-linear interpolation at grid interface

It is proven that numerical solution converges to a weak solution under certain conditions if a non-conservative interface algorithm is used

FIG. 4.3. Computed results of flowfield with a shock moving to the right and passing through a square grid.  $\rho_l = 3.948$ ,  $u_l = 4.359$ ,  $p_l = 5.005$ ,  $\rho_r = 1.4$ ,  $u_r = 3$ ,  $p_r = 1$ ,  $T = 0.13$ . Here subscript  $l$  and  $r$  indicate left and right side, respectively. (4.3) is adopted at the interface nodes, a scheme based on TVNI [18] is used within the each grid, and  $CFL = 0.95$ . (a) Mesh arrangement.  $G_A: 24 \times 24$ ,  $G_B: 18 \times 18$ . (b) Pressure contours for (a),  $\|S_{2\Delta}(0, T)\|_\infty = 2.4 \times 10^{-3}$ . (c) Mesh arrangement.  $G_A: 93 \times 93$ ,  $G_B: 69 \times 69$ . (d) Pressure contours for (c),  $\|S_{2\Delta}(0, T)\|_\infty = 7.1 \times 10^{-5}$ .

## II. Grid Interface Algorithm

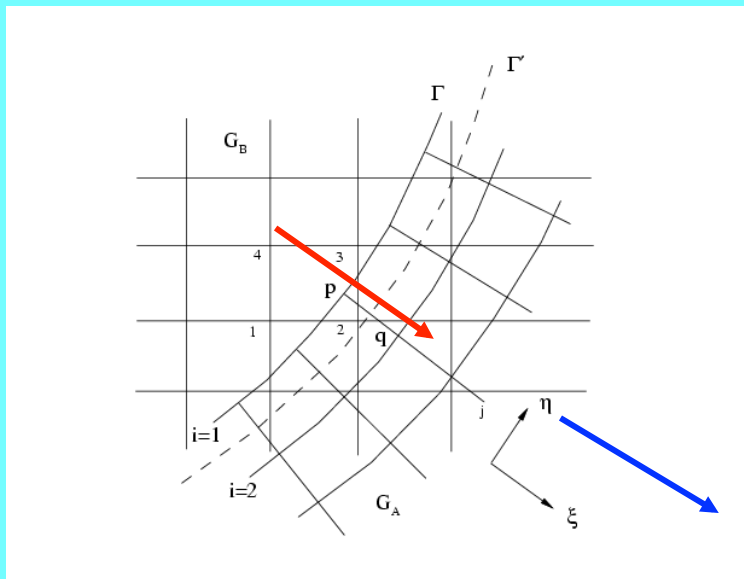
# Conservative or non-conservative treatment?



Non-physical solution obtained with linear interpolation

## II. Grid Interface Algorithm

# A mass conservation algorithm for incompressible flow



Implicit interface conditions  
implemented by Schwarz  
alternative iteration (1869)

An conservative interpolation

$$\sum_j \tilde{F}_{3/2,j}^1(U_p^A, U_q^A) \Delta \Gamma'_j = \sum_j \tilde{F}_{3/2,j}^1(U_p^B, U_q^B) \Delta \Gamma'_j,$$

### Algorithm 1

$$U_{1,j,k}^A = J_{1,j,k} \left( \left( \frac{U^I}{J} \right)_{1,j,k} + \left( \frac{U^I}{J} \right)_{2,j,k} - \left( \frac{U^A}{J} \right)_{2,j,k} \right),$$

$$V_{1,j,k}^A = V_{1,j,k}^I,$$

$$W_{1,j,k}^A = W_{1,j,k}^I,$$

$$p_{1,j,k}^A = p_{1,j,k}^I.$$

### Algorithm 2

$$u_{1,j,k}^A = u_{1,j,k}^I + u_{2,j,k}^I - u_{2,j,k}^A,$$

$$v_{1,j,k}^A = v_{1,j,k}^I + v_{2,j,k}^I - v_{2,j,k}^A,$$

$$w_{1,j,k}^A = w_{1,j,k}^I + w_{2,j,k}^I - w_{2,j,k}^A,$$

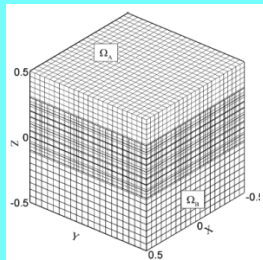
$$p_{1,j,k}^A = p_{1,j,k}^I.$$

Algorithm 1 and 2:

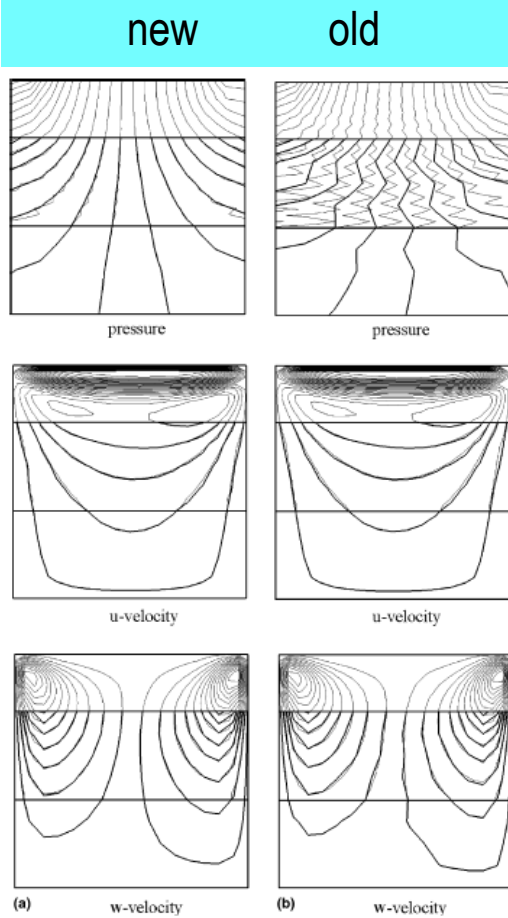
2<sup>nd</sup>-order accurate in conservation of  
mass and momentum fluxes

## II. Grid Interface Algorithm

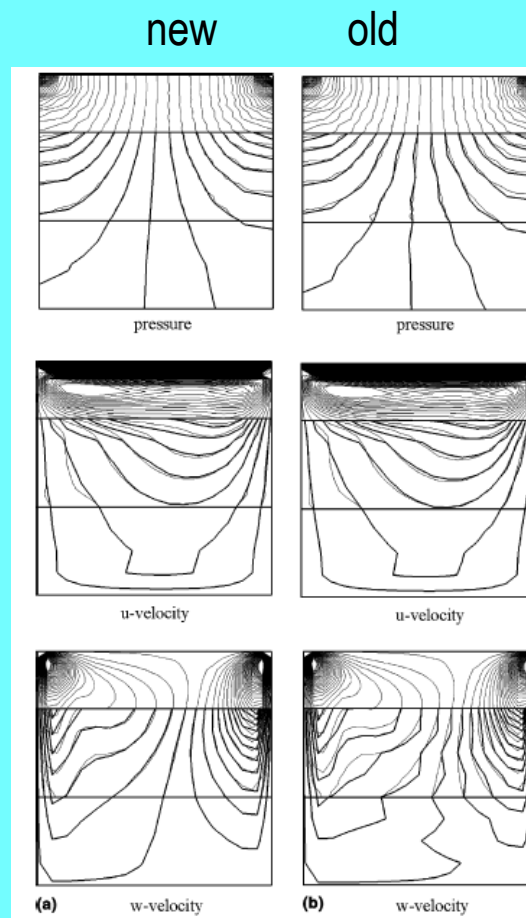
# Incompressible – numerical experiment



Oscillatory  
cavity flow

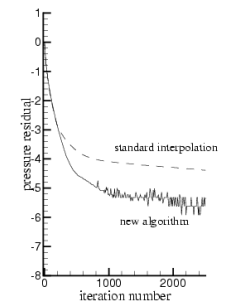


$t=0.25$

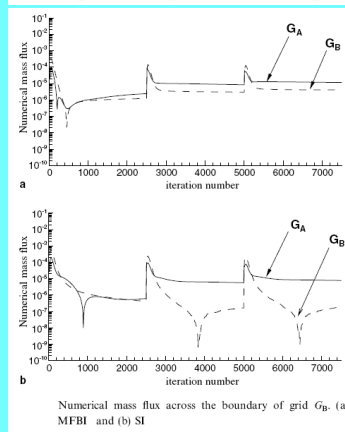


$t=10$

Convergence,  
conservation error



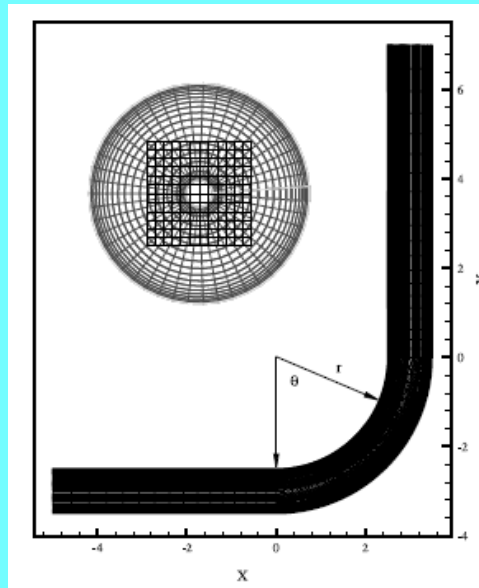
Convergence history for periodic cavity flow



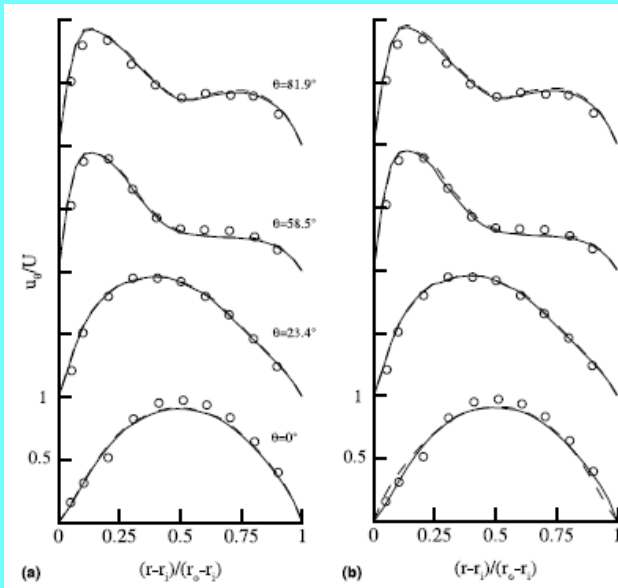
## II. Grid Interface Algorithm

# Incompressible – numerical experiment

Pipe bend flow



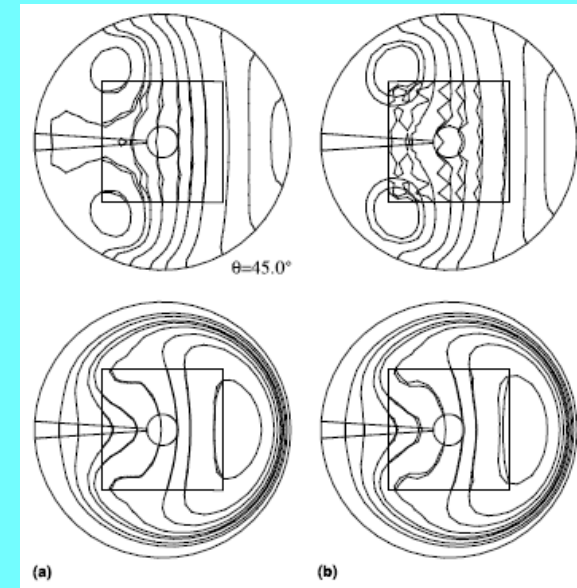
Horizontal symmetrical plane  
streamline velocity (computation  
– line, experiment – circle)



new

old

Cross section pressure (top),  
streamline velocity (bottom)



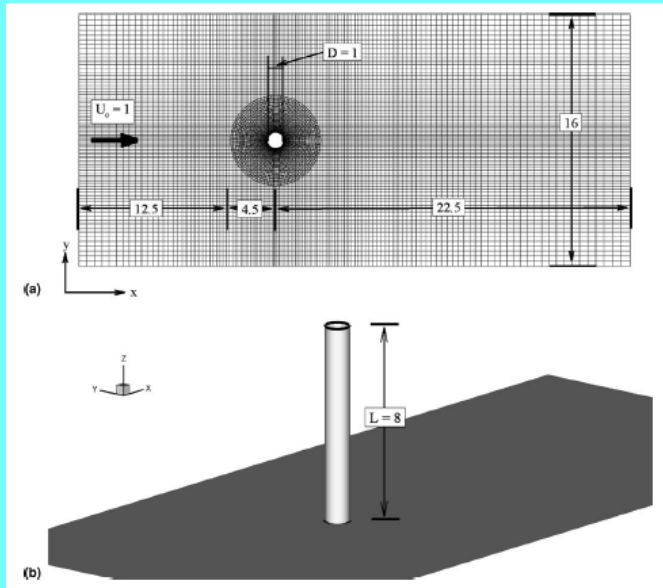
new

old



### III Incompressible Flow

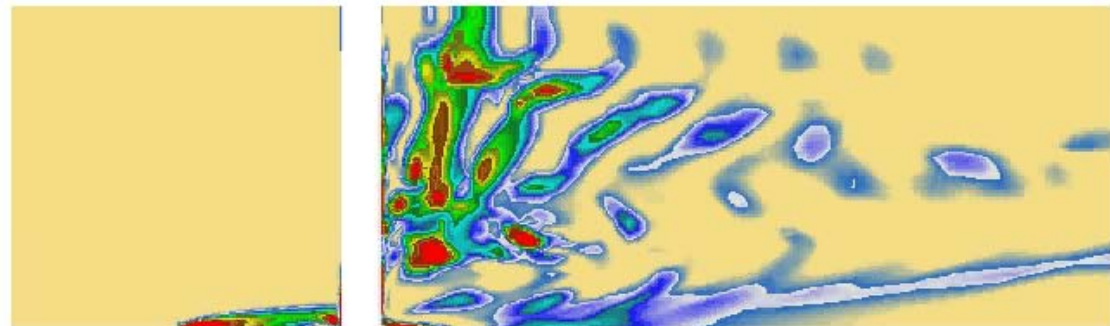
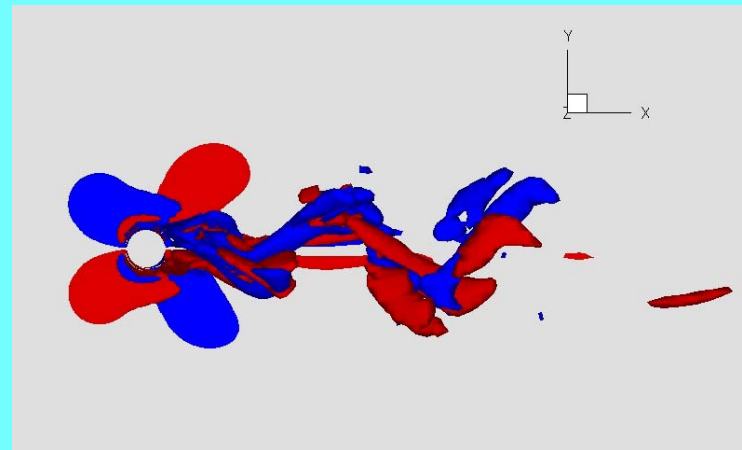
## Flow past cylinder – flow field



Configuration  
and meshes

$Re=300$

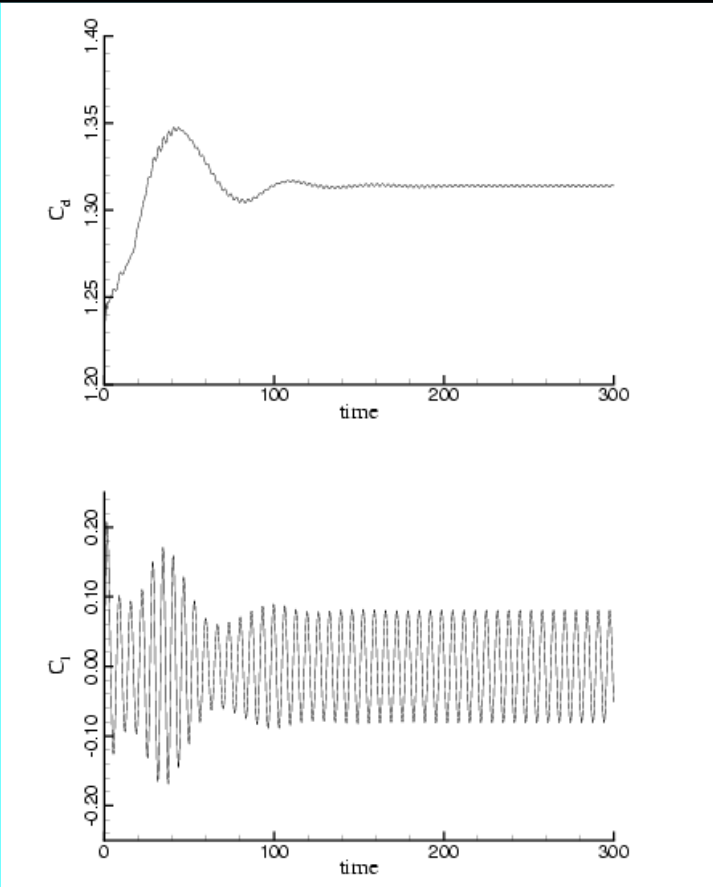
x-vorticity



y-vorticity

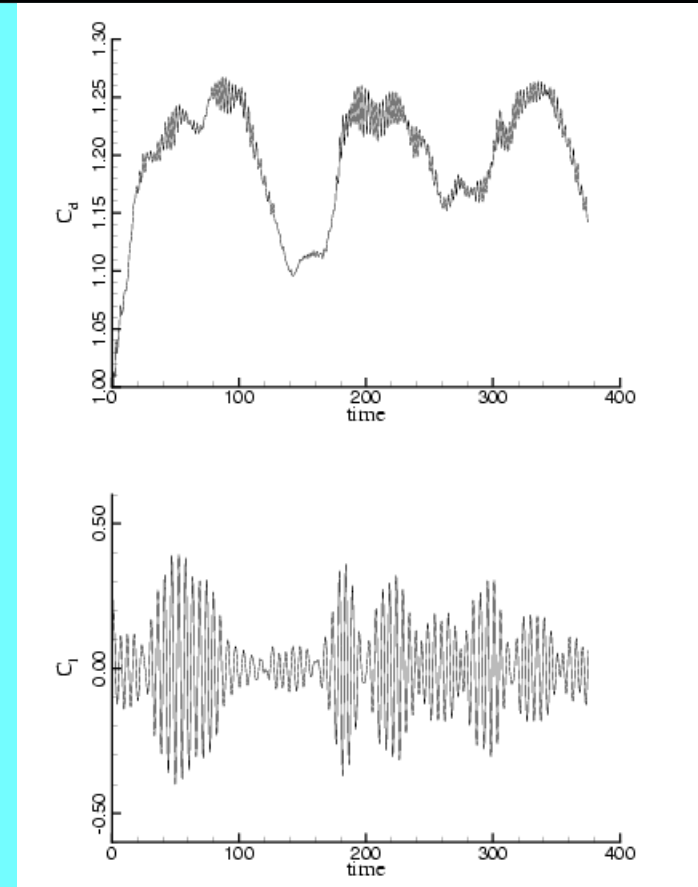
### III Incompressible Flow

## Flow past cylinder --- comparisons



**Re=100**

**St=0.152** Our result  
=0.145 Mittal, Phys. Fluids, v13, 2001  
=0.141 – 0.161 Norberg, F. Fluid Mech., v258, 1994

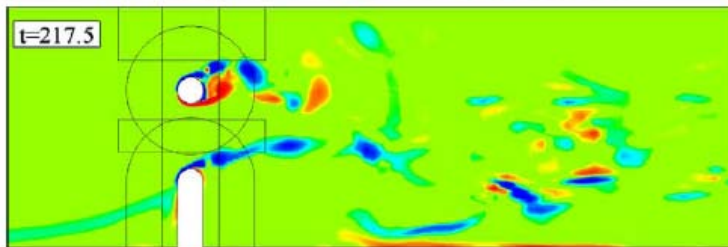
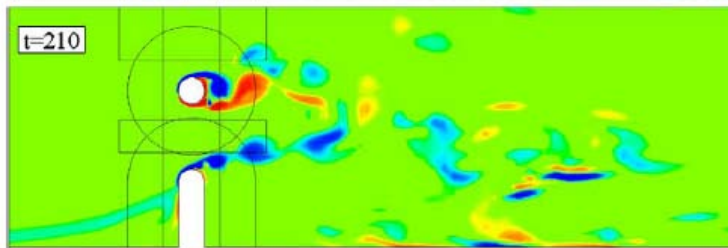
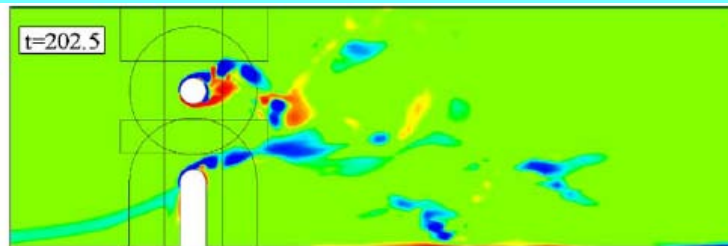
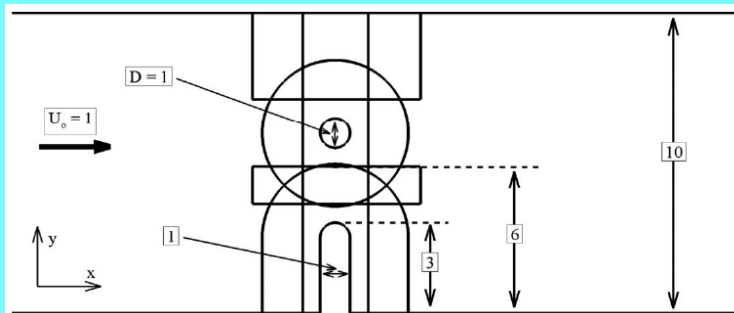


**Re=300**

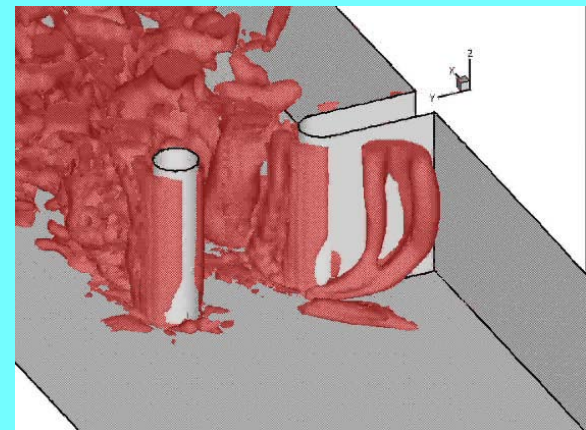
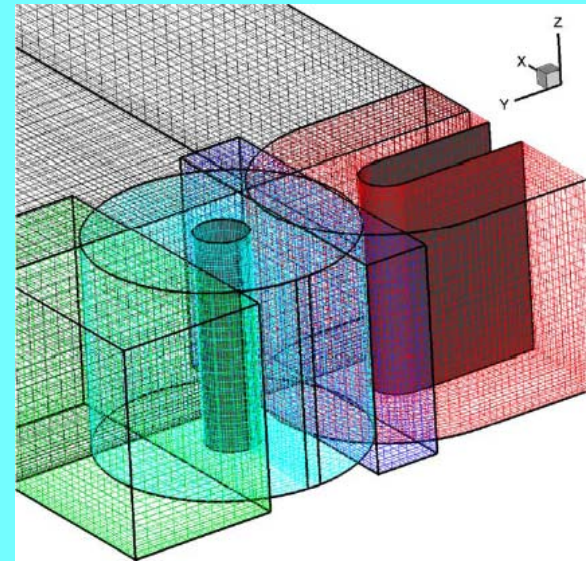
**St=0.195** Our result  
=0.190 Mittal, Phys. Fluids, v13, 2001

### III Incompressible Flow

## Flow past wall-mounted structures



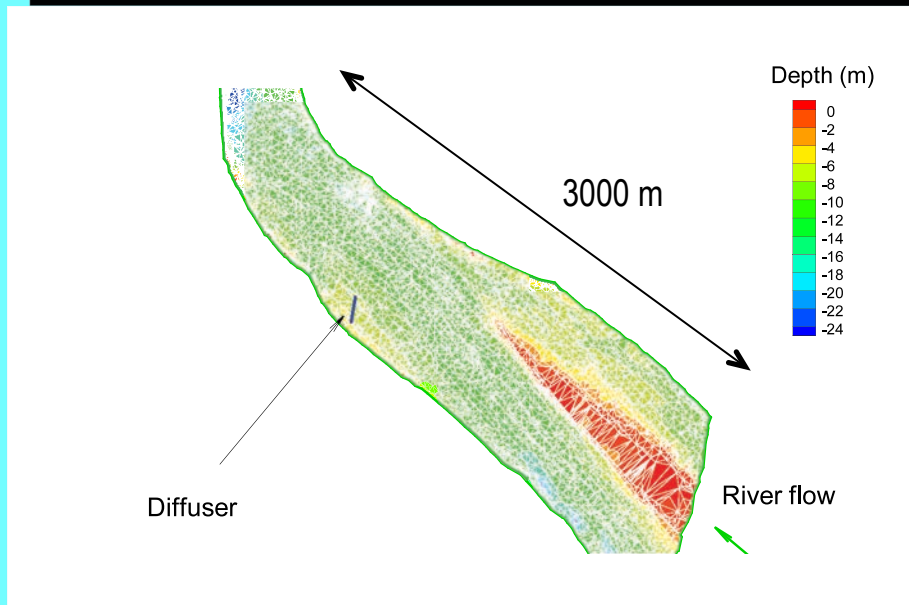
Re=300



3D vortex structure ( $\square$  2 iso-surface)

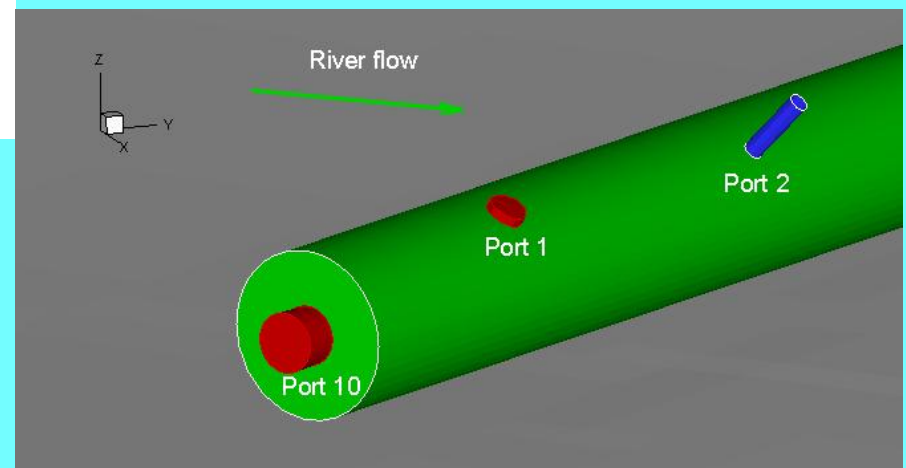
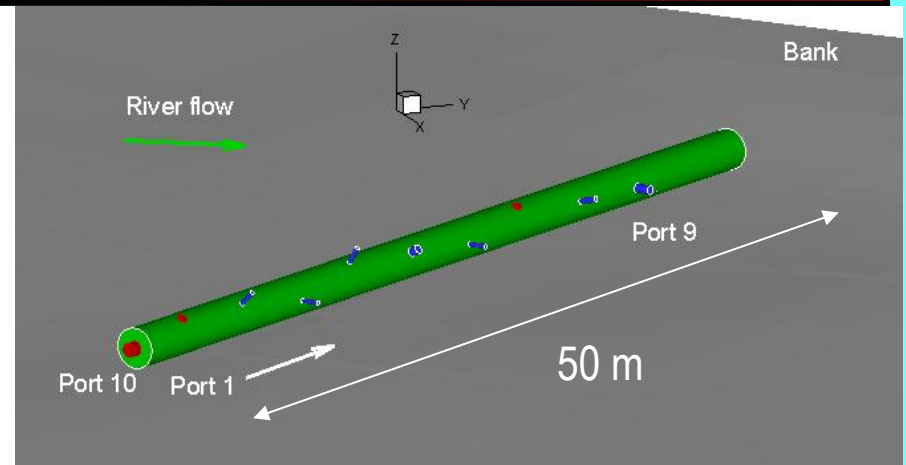
## IV Environmental Flow

# Thermal plume in natural river – flow parameter



bathymetry

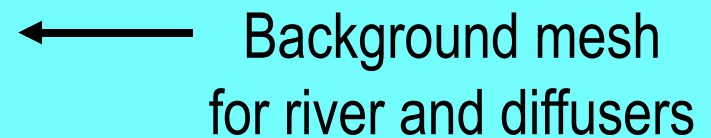
Ambient velocity: 0.5 m/s  
Effluent velocity: 2 -- 4 m/s  
Port diameters: 0.1—0.3 m



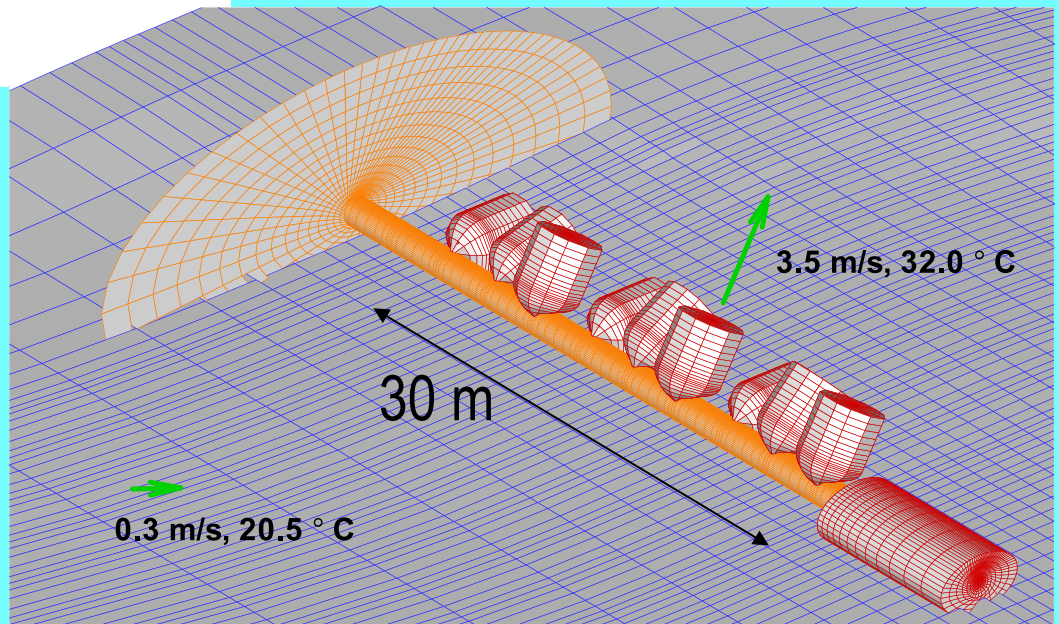
Close-up view of diffuser



# Thermal plume in natural river – Chimera overset grids

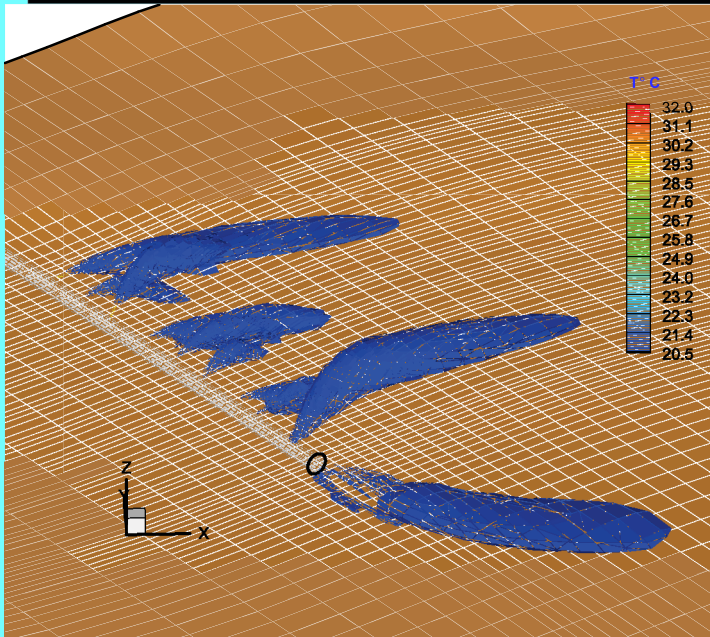


Total number of nodes  
300,000



## IV Environmental Flows

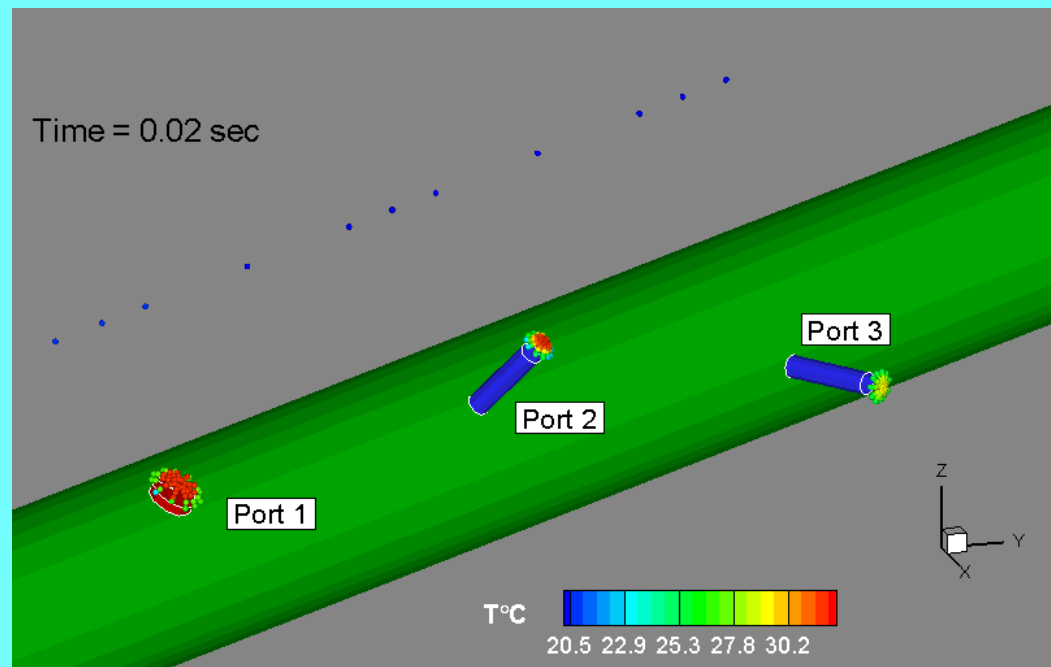
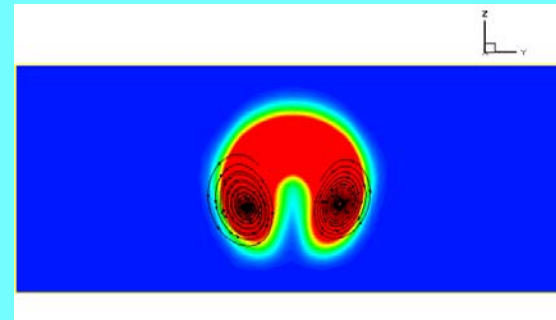
# Thermal plume in natural river – solutions



3D view of plumes  
(temperature iso-surfaces)

Particle tracking (movie)

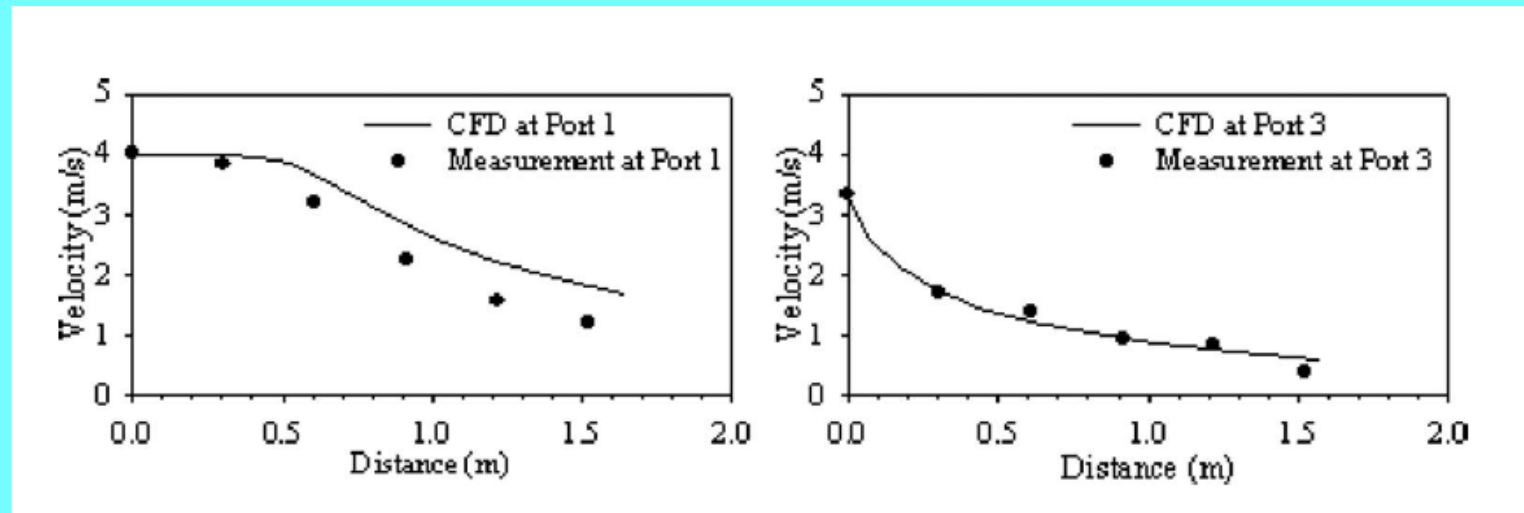
Individual plume



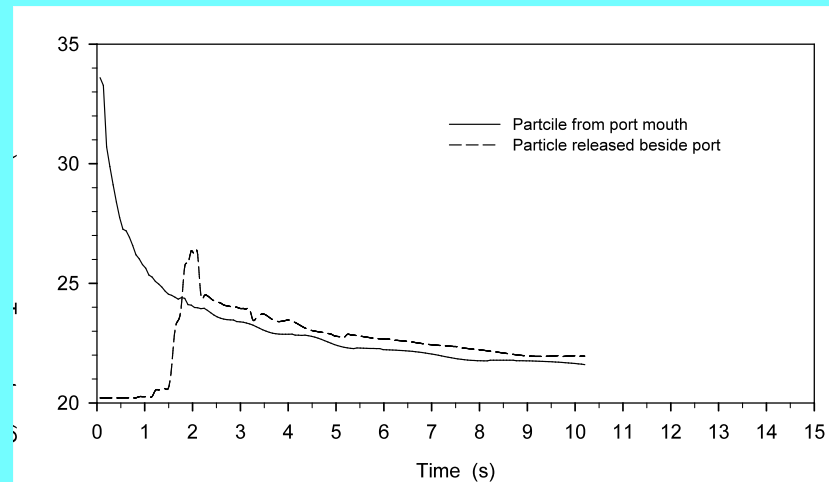
## IV Environmental Flows

# Thermal plume in natural river – predicted jets

Effluent velocity at port mouths



Time histories of particle temperature



# Background and models

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- Large scales -- Computational Geophysics Dynamics (GFD):  
 $O(10)\text{km} - O(10,000)\text{ km}$ ,  
 $O(1)\text{hr} - O(1)\text{ month}$

Note: Computational Fluid Dynamics (CFD):

Smaller scales:  $O(10)\text{ cm} - O(10)\text{ km}$

$O(1)\text{ ms} - O(1)\text{ hr}$

Individual phenomena: circulation, wave, etc.

- CFD model --- Unsteady, 3D, incompressible RANS, curvilinear coordinates, structured grids, finite volume method

FVCOM --- Unsteady, 3D, incompressible GFD equations,  
triangular mesh in horizontal direction, sigma coordinates  
in vertical direction, finite volume method



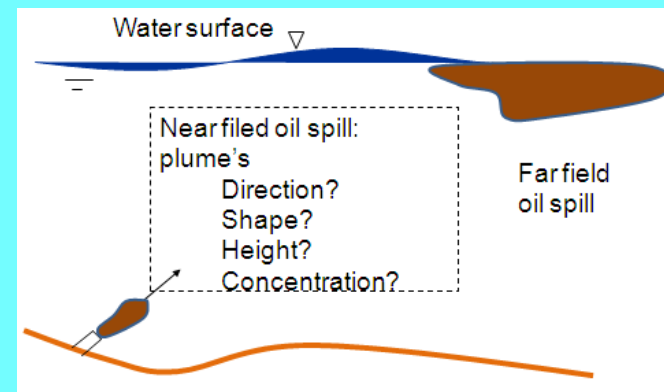
## V Coastal Ocean Flows

### Strategies -- examples

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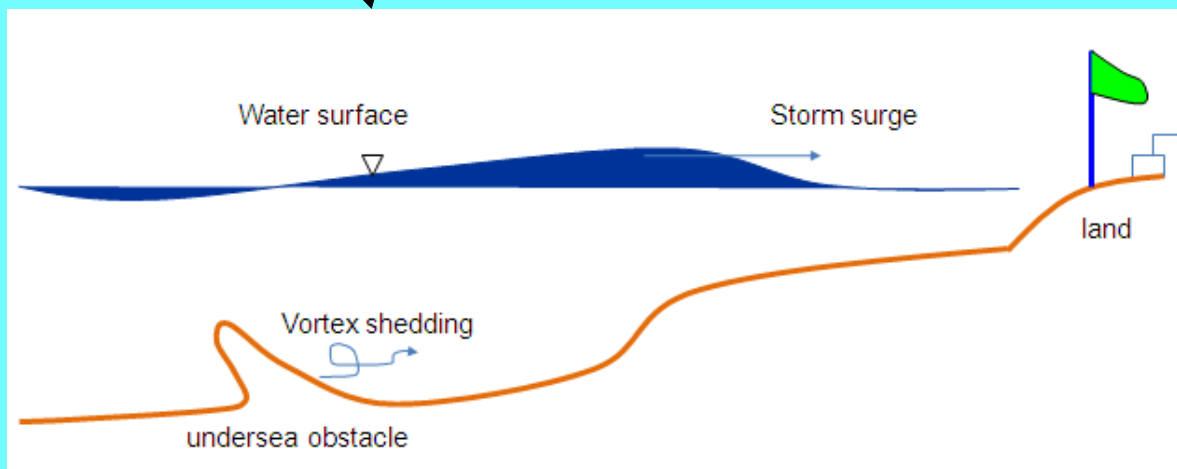
Discharge in ocean

- 1) small-scale initial plume of oil spill
- 2) large scale-floating dispersion



Coast process

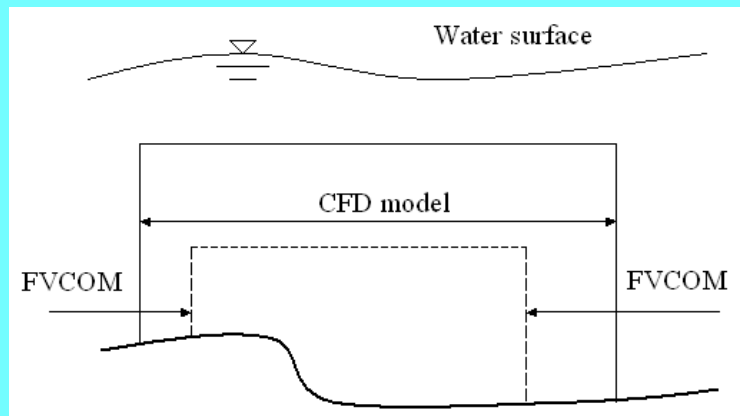
- 1) surge bore
- 2) underwater vortex flow



# Model coupling

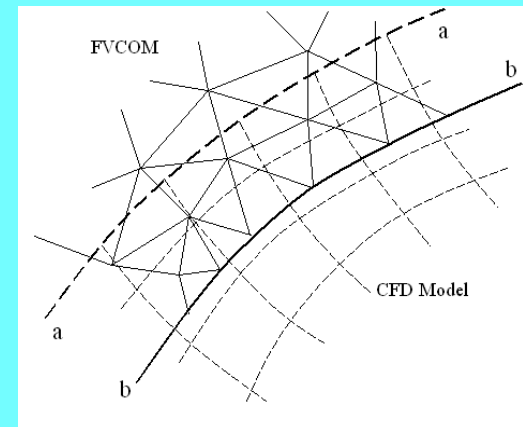
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Flow domain decomposition  
--- with overlap subdomains



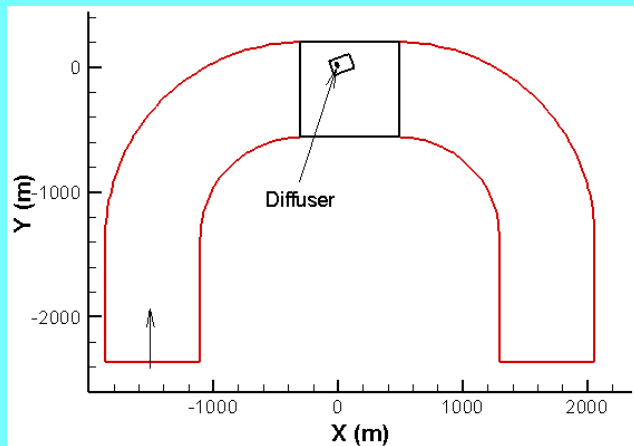
Coupling between CFD and internal mode of FVCOM --- exchange of solution for  $u$ ,  $v$ ,  $w$ ,  $p/\eta$

Chimera overset grids and Schwarz alternative iteration

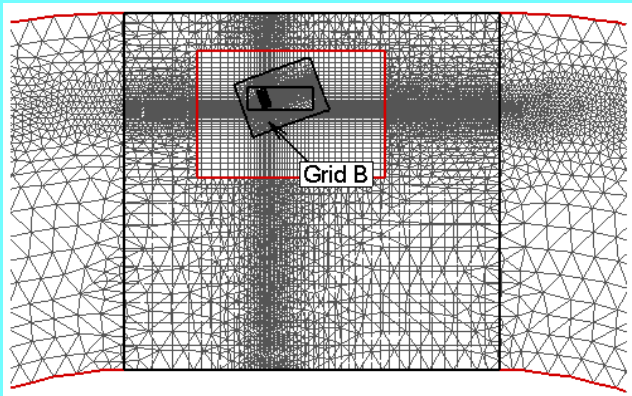


Second-order accurate interpolation at model interface

# Discharge into channel --- mesh

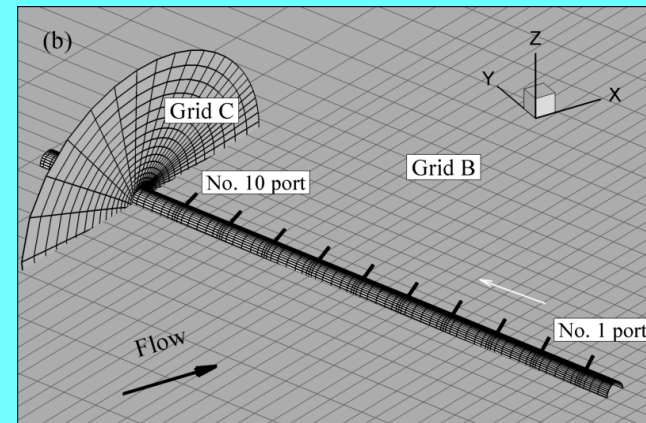


Channel and diffuser



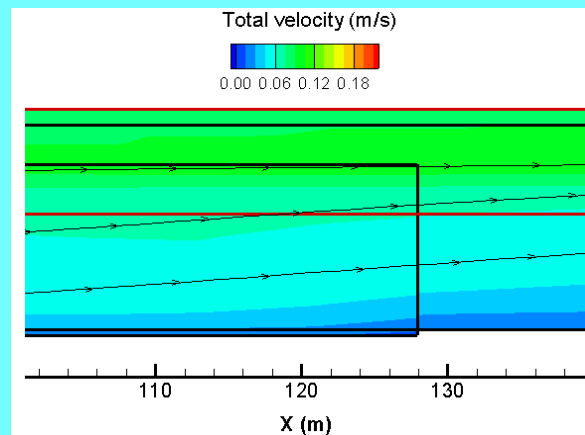
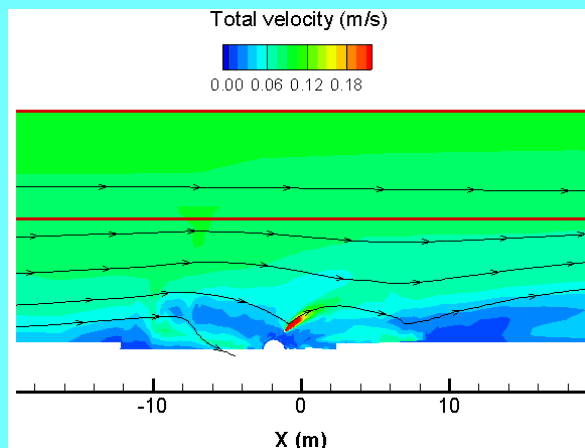
Mesh. Structured mesh – CFD  
unstructured mesh – FVCOM

Mesh:  
coupling – FVCOM: 115,000 nodes each  
layer, 11 layers  
CFD -- 220,000 nodes

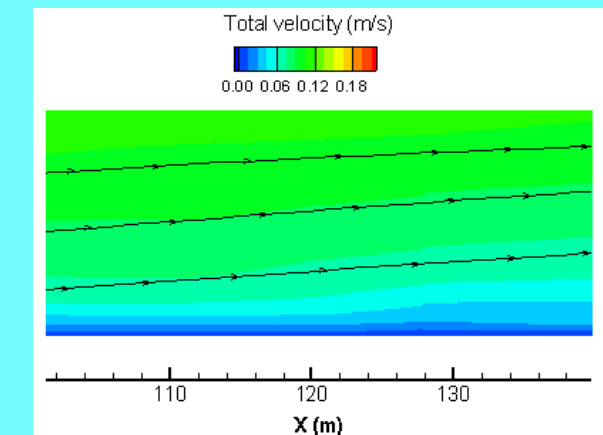
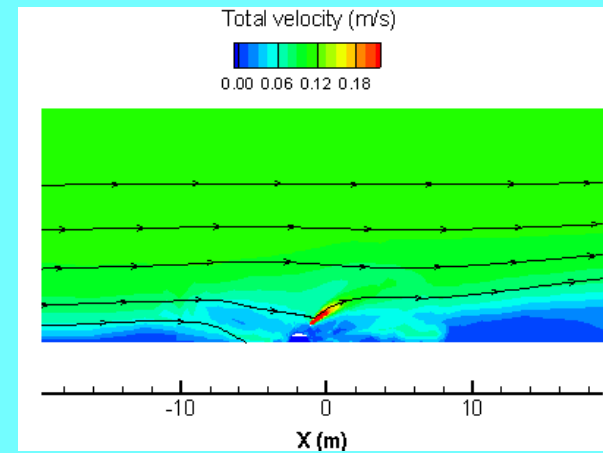


Mesh around the diffuser

# Discharge into channel --- results

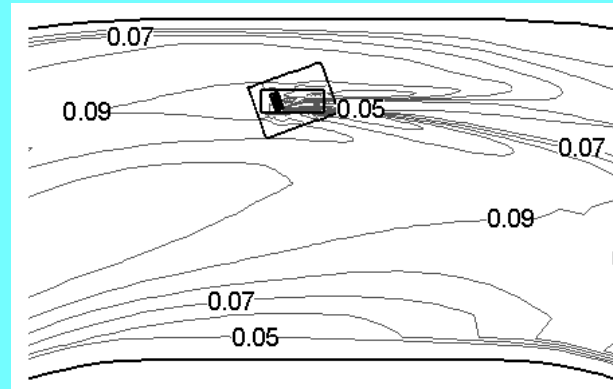
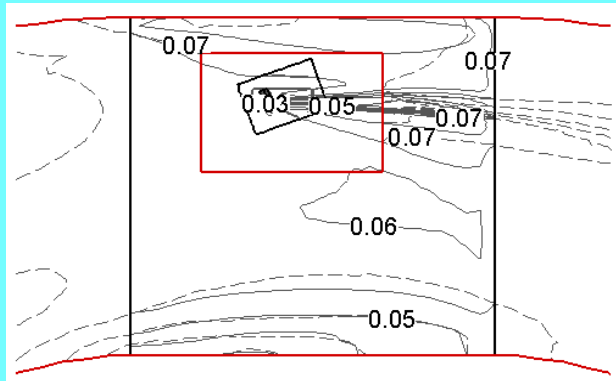


FVCOM/CFD coupling

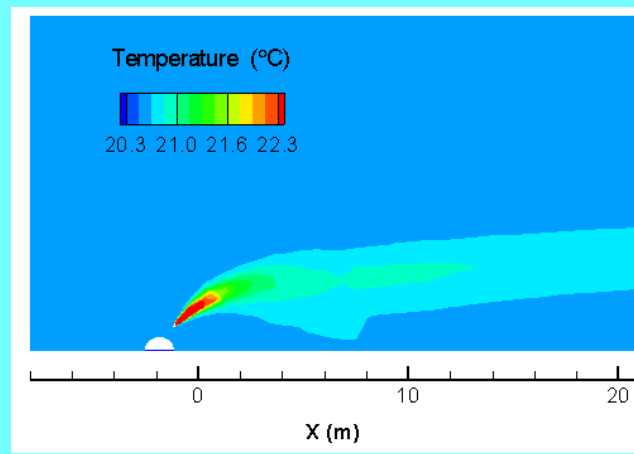
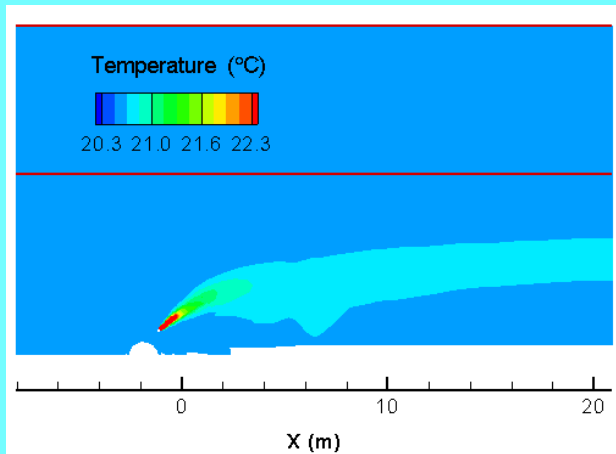


CFD

# Discharge into channel --- results



Total velocity  
(m/s)



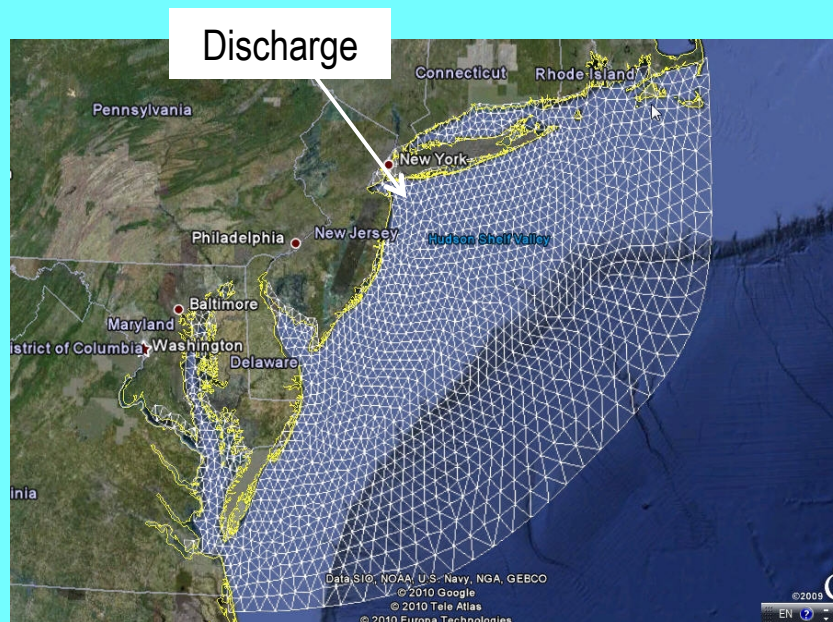
Temperature

FVCOM/CFD coupling

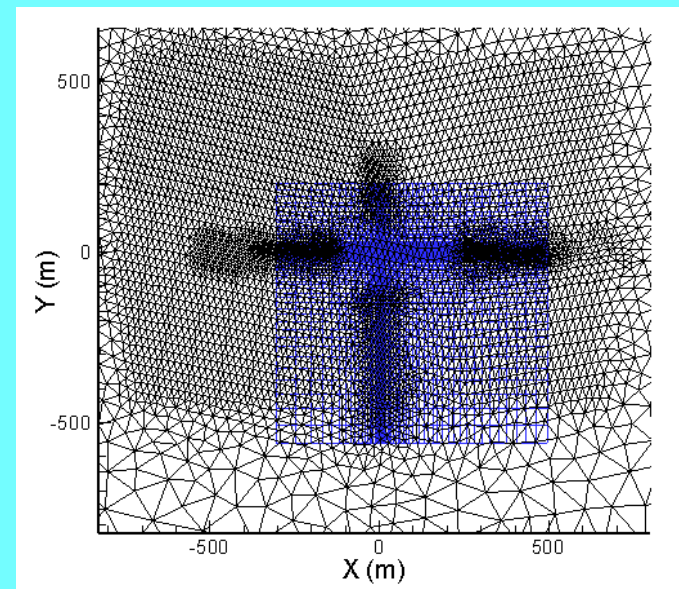
CFD

## V Coastal Ocean Flows

# Discharge into coastal flow --- mesh



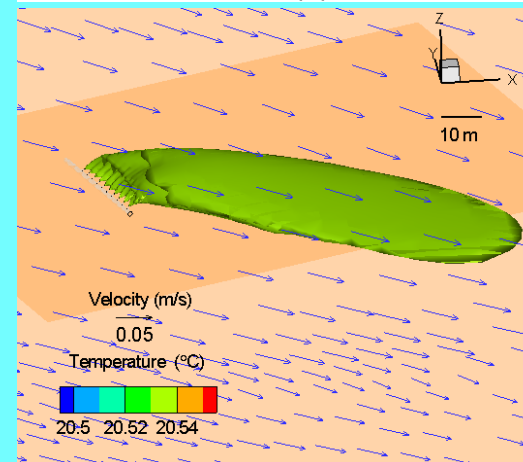
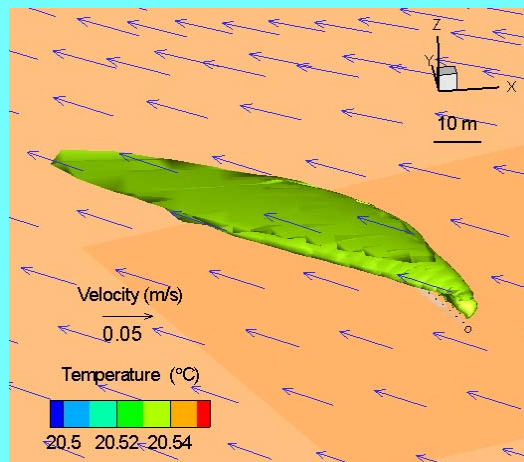
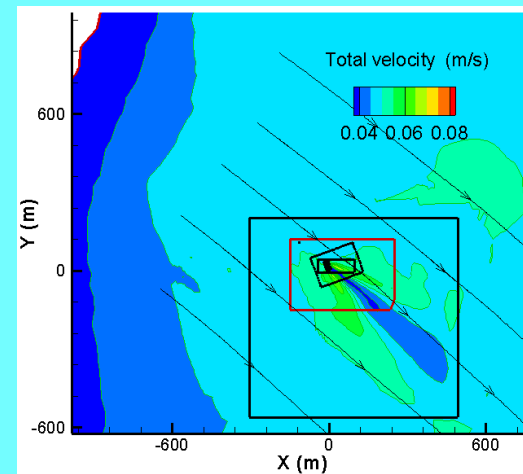
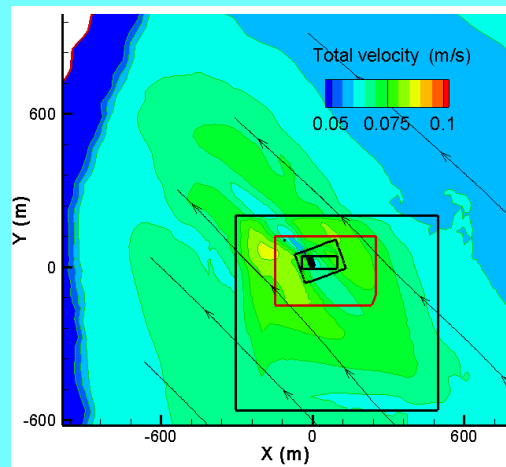
FVCOM mesh and CFD location



CFD mesh (blue)

New York/New Jersey coast region and FVCOM mesh and CFD location

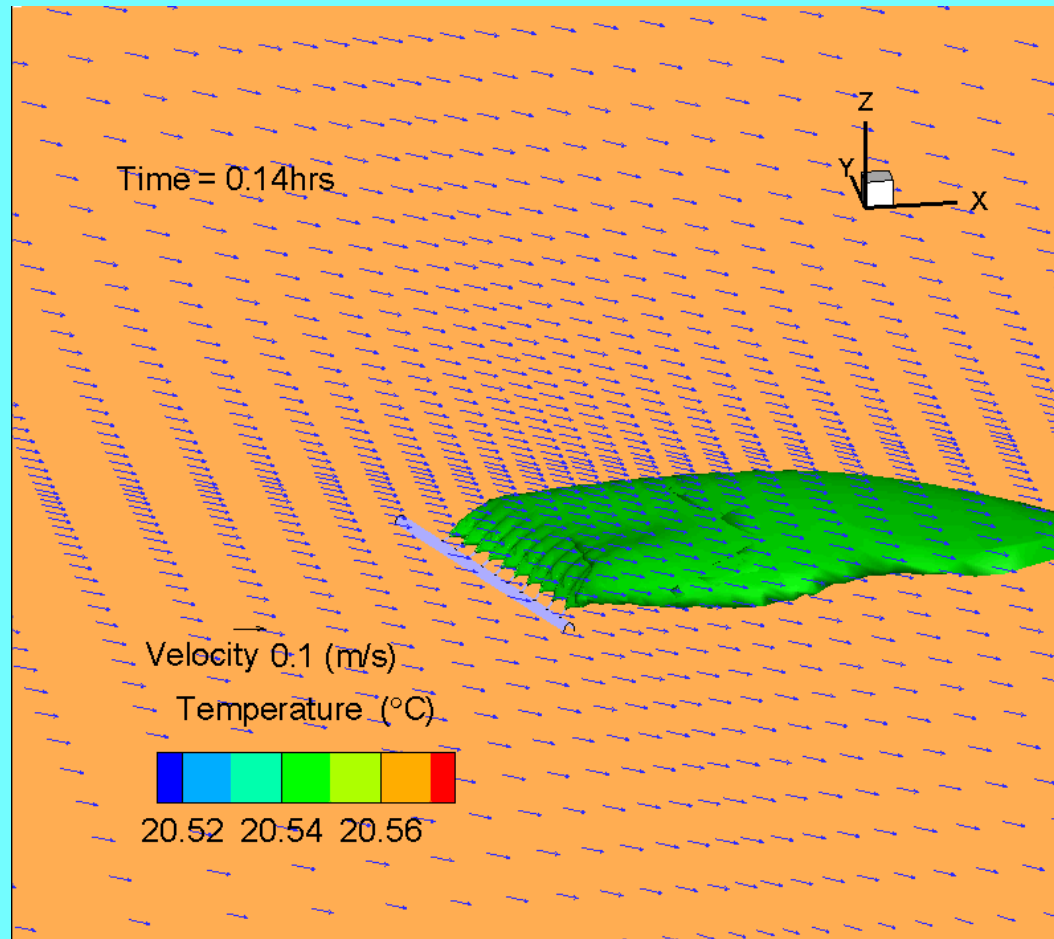
# Discharge into coastal flow --- solutions



Solution for thermal discharge. Top – velocity field, bottom – 3D thermal plume and water surface vectors, left – flood tide, right – ebb tide.



## Discharge into coastal flow --- solution movie



Animation of the temperature iso-surface under action of tides



# Concluding remarks

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## Conclusion and Future work

- 1) Overset grid techniques are powerful in resolving multi-scale and multi-physics problems
- 2) A systematic investigation on accurate and stable model interface algorithms is necessary
- 3) Challenges: coupling between different sets of PDE and flow models

## References

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----- Thanks -----